

# Logically Simple Objects and a Relational View of Reality in Wittgenstein's *Tractatus*, Russell and Carnap

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## Abstract:

Many philosophers have puzzled over the nature of the logically simple objects, the “substance” of the world, in Wittgenstein's *Tractatus logico-philosophicus* (*TLP*). Such questions are misplaced because *TLP* is committed to the view that talk of such metaphysically problematical entities is part of the “ladder” that must be “thrown away” after one has climbed it. Further, *TLP*'s demotion of its logically simple objects to mere logical subjects requires an *increased emphasis* on the relations between these alleged objects. *TLP*'s account of its logically simple objects is an application of Russell's *relational* view of mathematical objects from his *Principles of Mathematics* applied to “reality”. Carnap develops an analogous relational or “structural” view of reality in *The Logical Structure of the World*. Despite important differences between them, these three philosophers can profitably be seen as replacing the traditional emphasis on substances with a *relational model of reality*.

**Keywords:** Tractatus-logico-philosophicus, logical simples, relationalism, Russell, Wittgenstein, Carnap

We can describe the world completely by means of fully generalized propositions, i.e., without first correlating any name with any particular object.

Wittgenstein, *Tractatus* (5.526)

What are the logically simple objects (hereafter LSO's) posited by the *Tractatus logico-philosophicus* (*TLP*) and *Notebooks 1914-16* (*NB*) (Black 1961, 49; Tejedor 2015; Proops 2022, § 4).<sup>1</sup> Could a watch or visual patch be an LSO? (*NB*, 62-66) Do LSO's have an essential character? Could a property or a relation be an LSO? Can we become *acquainted* with LSO's or are these merely theoretical requirements of logic? In fact, these questions are misplaced. *TLP* is committed to the view that talk about LSO's and about their fully analyzed names is eliminable in a logically adequate language. This may seem surprising because LSO's are said to be the “substance” in the world (2.021, 2.027). However, *TLP*'s talk of such metaphysical entities is part of the “ladder” that must be “thrown away” after one has used it to “see the world right” (6.54). Questions about the nature of LSO's and their unanalyzable names evaporate once one understands *TLP* correctly. However, the *elimination* of LSO's means that

the most significant contribution to the sense of *TLP's* fully analyzed elementary propositions rests on the *relations* in which LSO's are represented. (2.0231) The paper also argues that *TLP's* account of LSO's is an application of Russell's *relational* view of mathematical objects formulated in *Principles of Mathematics (POM)* applied to the real world and explains an illuminating analogy between *TLP's* relational view of reality and Carnap's view in *The Logical Structure of the World (LSW)*.

### I.) Simple Objects, Substance and Possible Worlds

Logic ... is prior to the question 'How?', not prior to the question 'What?'

Wittgenstein, *Tractatus* (5.552-5.5521)

According to *TLP* the logic of the world, i.e., the limits and structure of logical space, is determined by the ground-level set of LSO's. Since it is natural to assume that, for *TLP*, the actual world is composed of an *enormous* number of LSO's that, in turn, may or may not be configured in an enormous number of ways, it is useful, for heuristic purposes, to present a simplified model of the connection between the LSO's and the limits of logical space. If Norman Malcolm is composed of 27 trillion LSO's involving 9 trillion different possible relations this would generate an unmanageable enormous number of different possible worlds. Consider, therefore, a fictional world in which there are far fewer LSO 's and possible relations and, therefore, a far smaller set of possible facts and possible worlds. Since "there are no privileged numbers" (*TLP* 5.453), one cannot rule out that there could be only a tiny number of LSO's, perhaps even just two objects and one possible relation. One can, therefore, describe the list of possible worlds there would be if there were only two LSO's, a and b, and one possible relation, R, in which they can participate. Assume further that this relation R is symmetrical but not reflexive. That is, if a can be R to b then b can be R to a, but neither a nor b can be in relation R to themselves. Assume also the principles of classical logic: There are no contradictory states of affairs, e.g., no possible world in which a is both R to b and not R to b. This yields only 4 possible worlds (PW's),

PW1: a is R to b; b is R to a  
PW2: a is R to b; b is not R to a  
PW3: a is not R to b; b is R to a  
PW4: a is not R to b; b is not R to a

Fig. 1.

Although *TLP's* LSO's are not familiar objects like Bob and Mary, it is useful to sketch an everyday language interpretation of the model in Fig 1. Suppose that "a" = "Bob", "b" = "Mary", and "R" = "loves". Thus, in PW1 Bob loves Mary and Mary loves Bob. In PW2, Bob loves Mary and Mary does not love Bob. In PW3 Bob does not love Mary but Mary loves Bob. In PW4 Bob does not love Mary and Mary does not love Bob.

Assuming also that one represents the proposition that a is R to b by the propositional symbol “aRb” and, employing the usual truth functional connectives, *TLP*'s descriptions of these four possible worlds (DPW's) is given in Fig. 2,

DPW1: “aRb & bRa”  
 DPW2: “aRb & ~bRa”  
 DPW3: “~aRb and bRa”  
 DPW4: “~aRb & ~bRa”

Fig. 2.

Since, as *TLP*(5.526) states, one can symbolize such propositions in completely general form using quantificational logic without correlating any name with any object, one can completely represent the possibilities in Fig. 2 in the fully general propositions in Fig. 3,

DPW1:  $(\exists x)(\exists y) (x \neq y \ \& \ xRy \ \& \ yRx)$   
 DPW2:  $(\exists x)(\exists y) (x \neq y \ \& \ xRy \ \& \ \sim yRx)$   
 DPW3:  $(\exists x)(\exists y) (x \neq y \ \& \ \sim xRy \ \& \ yRx)$   
 DPW4:  $(\exists x)(\exists y) (x \neq y \ \& \ \sim xRy \ \& \ \sim yRx)$

Fig. 3.

Following 5.526, in Fig. 3 one says everything said in Fig. 2, thereby representing the four possible states of reality, without correlating any name with any object. One might, however, argue that these four descriptions in Fig. 3 leave out the most important features of these PW's. For, although DPW1-DPW4 do capture the abstract *formal structure* of these four possible descriptions and the corresponding four PW's described in Fig's 1 and 2, these completely generalized descriptions in Fig. 3 *leave out Bob and Mary*. For, when, in everyday language, one is told that Bob loves Mary one is not merely told that there are two distinct objects where the one loves the other one. One is told that a specific individual, Bob, loves another specific individual, Mary. Those two concrete individuals, Bob and Mary, are the actual subject matter in the world that determines the sense or content of these propositions. By translating the four propositions in Fig. 2 into the fully generalized forms in Fig. 3 it appears that one loses the most important linguistic devices, the names that represent the subject matter in the world and, with that, the most important content of the propositions in Fig. 2, the two real subject matters of the propositions, Bob and Mary.

It is highly significant that though this seems true for everyday language, it does *not* hold for *TLP*'s completely logically analyzed language. *TLP*'s model of the role of LSO's and their possible configurations in the world is virtually the *opposite* of the role names and their references have in everyday language. In order to see this, one must look more closely at the peculiar nature *TLP*'s LSO's.

## II.) A Geometrical Model of *Tractatus* Logical Space

If two [simple] objects have the same logical form, the only distinction between them apart from their external properties is that they are different.

Wittgenstein, *Tractatus* (2.0233)

One cannot imagine anyone claiming of ordinary everyday objects like Bob and Mary that the only distinction between them apart from their external properties is that they are different. Perhaps Bob is distinguished from Mary in that one of Bob's "external properties" is that he always wears blue and one of Mary's "external properties" is that she always wears red, but it is not *only* by means of "external properties" that Bob and Mary are distinguished. For, at least on the everyday understanding, objects like Bob and Mary have all sorts of *internal* (intrinsic) properties that distinguish them, e.g., Bob is male and Mary is female.

One might argue that the everyday distinction between external and internal properties is vague, e.g., that Bob could transition to female, suggesting that gender is also an external property. However, we *do* make the distinction between external and internal properties in ordinary language and Bob's O-negative and Mary's A-positive blood would be almost impossible to change without causing their demise. That is, in *ordinary language* we assume that if an object can have external properties, it must be a determinate object in its own right with its own internal nature in order to be the *substrate* for its external properties. It appears, therefore, that in ordinary language the notion of an object's internal properties is conceptually prior to that of its external properties. Russell says something like this about numbers at *POM* (§ 242): "If [numbers] are to be anything at all they must be intrinsically something." In any case, the present paper argues that this does *not* hold for *TLP* objects and, on that basis, develops a crucial contrast between the ordinary conception of an object and *TLP's* novel concept of LSO's.

Since ordinary objects do not resemble the LSO's described at *TLP*(2.0233), one wonders if there is any actual system of items that satisfies that description and, in fact, there is (Goddard and Judge 1983). Consider the ordinary concept of the Euclidean plane as an ordered system of points in which each point bears a determinate relation to each of the other points in the plane. These Euclidean points are, however, very unusual objects. For each point in that plane, taken by its self, is completely indistinguishable from every other point in that plane. Employing the Cartesian coordinate system, the point (2, 4) at 2 on the x-axis and 4 on the y-axis is, in itself, completely indistinguishable from the point (4, 9) located at 4 on the x-axis and 9 on the y-axis. One *can* distinguish them but only by putting some *external* marks at these points. Since one cannot distinguish these two pure geometrical points by any of their intrinsic properties, *because they have no intrinsic properties*, one can only distinguish them by, so to speak, putting something, i.e., a mark, at those points on the graph.

This geometrical analogy may help explain why *TLP* (2.023) states that "In a manner of speaking objects are colorless". For the points in the Euclidean plane, considered in themselves, are *literally* colorless (and soundless, weightless, etc.) which is why, in order to distinguish them we must

use some external mark, an ink mark or a mathematical notation like “(2, 4)”. This is connected with *TLP*(3.221), one of *TLP*’s mysterious mystical, remarks,

Objects can only be *named*. Signs are their representatives. I can only speak *about* them. I cannot *put them into words*. Propositions only say *how* things are, not *what* they are [all emphasis, LW’s].

*TLP*’s fully analyzed propositions cannot express the individual nature of the LSO’s they describe, if LSO’s even have any inner nature or “whatness.” Since *TLP*’s LSO’s resemble the points in the pure Euclidean plane that are, in themselves, internally indistinguishable from each other, it committed to hold that one cannot meaningfully talk about their nature (*TLP*, 6.5). Thus, *TLP*’s fully analyzed elementary propositions can only describe the *relations* between these internally indistinguishable LSO’s, i.e., *how* they are configured.

*TLP* may describe its LSO’s as substances but on this geometrical model they do not resemble objects or substances traditionally conceived. Just as it makes no sense to talk about replacing one point in pure geometrical space, e.g., the point “(2, 4)” in a Cartesian coordinate system, with the point “(4, 9)” in that space, it makes no sense to talk about replacing one *TLP* LSO at some point in logical space with another from a different position in logical space. Just as a point in pure Euclidean space is individuated solely by its place in *that relational system*, *TLP*’s LSO’s are also individuated only by their position in the system of *relations* that constitutes logical space. Since a *TLP* LSO  $O_1$  is individuated by its position in that relational system, any object occupying that position in logical space just *is*  $O_1$ , not a different object that had somehow been substituted for it. This is concealed if one thinks of *TLP*’s LSO’s as if they are substantial entities like Bob and Mary with their own intrinsic identities that might be moved around from here to there.

One might profitably compare *TLP*’s LSO’s to the “it” in the everyday statement, “It’s raining”. For that sentence to be true” there need be no actual object in the world, the reference of “it”, that does the raining. To say “It’s raining” is, roughly, only to say that there is a complex set of relations in the world that constitute rain and that could, in principle, be described by a physicist. Just as the *syntax* of ordinary English demands a subject for the predicate, an “it” to do the raining, even though no such substantial “it” need exist in the real world, *TLP* holds that logic demands a subject for its fully analyzed relations, but this demand is *purely formal*: “The substance of the world *can* only determine a form” (2.0231).

Since *TLP*’s LSO’s resemble the internally indistinguishable points in Euclidian plane, and since the only way simple objects enter into the meaning of propositions about the world is as formal placeholders for the relations, *TLP*’s fully analyzed propositions need only distinguish LSO’s *formally*, e.g., if the facts about the world, as described in DPW2 from Fig 1 in § I are that a is R to b and that b is not R to a, one can describe this possible world completely, leaving out *nothing*, by the fully generalized proposition DPW2 from Fig. 3,

$$(\exists x)(\exists y) (x \neq y \ \& \ xRy \ \& \ \sim yRx)$$

If the objects over which “x” and “y” range are like Bob and Mary then something *is* left out by this proposition, namely, Bob’s and Mary’s intrinsic natures. Since, however, *TLP*’s simple objects have no intrinsic natures, *nothing* is left out by DPW2. It is true that the objects ranged over by “x” and “y” in DPW2 are as different as the two distinct geometrical points (2, 4) and (4, -1), but since these points are not distinguished by their internal properties they can only be distinguished formally, but that is accomplished by DPW2 in Fig. 3.

Since *TLP*’s simple objects appear to be formal requirements of logic, one might think that both they and logical space are *transcendentally ideal*. However, although *TLP* (6.13) does say that “logic is transcendental,” it nowhere suggests that simple objects are *created* by the mind. Rather, since *TLP*’s LSO’s “only determine a form”, the names for these “objects” are completely replaceable by purely formal logical apparatus that, according to *TLP*’s “fundamental idea” (4.0312), do not stand for anything. Thus, even if God miraculously understands the language that, up to that point, following *TLP*’s “solipsism” (5.62), I alone had understood, it would still be true that the substance of the world “only determines a form”. Therefore, God would only understand *the same empty form that I do!* One should, therefore, say that *TLP*’s simple objects and logical space are formal “presuppositions” (6.124) of logic much as the system of different but internally indistinguishable geometrical points are a presupposition of Euclidian geometry. There is nothing in the least bit subjective or idealistic about that. Wittgenstein does talk about “my world” but “never speaks of ‘my logic’” (Black 307). Given that this geometrical model lies at the foundation of *TLP*’s view of the substance of the world, *TLP* is the most thorough-going attempt up to that time to defend a version of the pure geometrical model of the world initiated by Plato (Cf. Popper 1965, 88).

### III.) The Relational Character of Reality

The substance of the world can only determine a form, not any material properties. For it is only by means of propositions that material properties are represented [*dargestellt*], only by the configuration of objects that they are produced [*gebildet*].

Wittgenstein, *Tractatus* (2.0231)

Whereas § II discusses the fact that *TLP*’s LSO’s and logical space resemble, respectively, pure geometrical points and pure Euclidean space, *TLP* also sketches an account of the “production [*bilden*]” of the material world. That is, *TLP* does *not*, on the geometrical interpretation, hold that its simple objects are indistinguishable but only that, like the empty points in the Euclidean plane, they are not distinguishable by virtue of their “internal properties” but only by their contingent configurations.

Consider again the simple world sketched in § I in which there are only two simple objects, a and b, and one possible relation R, yielding four possible worlds. Recall from Fig. 2 that in the first of these possible worlds PW1, a is R to b and b is R to a. In the second, PW2, a is R to b and b is not R to a. In the third, PW3, a is not R to b and b is R to a. In the fourth, PW4, a is not R to b and b is

not R to a. Suppose further, for heuristic purposes, that when in PW2 a is R to b and b is not R to a, the world has the material property of a blue glow and that when in PW3 a is not R to b and b is R to a, the world has the material property of a red glow. On these assumptions there is a sense in which a and b, even though internally indistinguishable, can be distinguished. For a is distinguished by virtue of the fact that its being is R to b when b is not R to a “produces” the material property of a blue glow. What does it *mean* to say that a is R to b while b is not R to a? “Can’t you see the blue glow?”

*TLP* (2.0233) does not, therefore, hold that LSO’s, as completely indistinguishable in themselves as two empty points in geometrical space, cannot be distinguished. It holds that different LSO’s can only be distinguished by virtue of their contingent configurations. On *TLP*’s model of reality the “material properties” of the world, the blue glows, are “produced”, not by the “substances” of the world (that “only determine a form”), but by their configurations. This is a model of the reality on which the LSO’s function only as formal subjects for the all-important relations. For the existence of these relations produce the “material properties”. Thus, for *TLP*, the everyday proposition, e.g., that the world exhibits a blue glow, must be analyzable into a conjunction of elementary propositions about the contingent relations between the LSO’s that are, in themselves, completely indistinguishable.

This also suggests an interesting interpretation of *TLP*’s (2.0251) puzzling remark that “Space, time and color (being colored [*Färbikeit*]) are forms of objects.” Note that space here *cannot* mean the logical space discussed in the previous section. *TLP* is here referring to the “material properties”, in a broad sense, of physical space, time and color in the actual world. Since “form is the possibility of structure” (2.033), to say that space is the form of the LSO’s is to say that the spatiality of the actual world is “produced” by a certain combination of the LSO’s. Similarly, physical time is “produced” by a different combination of the LSO’s. Finally, the color in the world is produced by combining LSO’s yet in another way.

It is a corollary of this that the world of LSO’s, substances, considered in itself, is devoid of time, space or color. *TLP* does not envisage a set of simple substances in physical space and time prior to entering into any actual configurations. The physical phenomena of space, time and color are each “material properties” that are “produced” by the configurations of LSO’s. This should not be surprising. The logical point of view is the point of view on the logical possibilities, that is, on logical space as such. But that is to view the word “*sub specie aeterni*” (6.45), i.e., prior (logically not temporally) to any definite configuration in LSO’s that make up the “material properties” of the world.

#### IV.) An Analogy with Carnap’s Logical Positivism

The actual basic concepts of the constructional system ... to which all other concepts are to be reduced are not the basic objects but the basic relations [because the] system of relations is primary relative to its members.

Carnap, *The Logical Structure of the World* (§ 7)

Although Carnap took *TLP*’s logical system as the template for his views in *LSW* there are

important differences between the two. Perhaps the most important is that Carnap (1935, § I.7) sees *TLP* as “inconsistent” because it says that “whereof one cannot speak thereof one must be silent; and then, instead ... he writes a whole philosophical book.” Carnap aims in *LSW* to produce a logical-philosophical system modeled on *TLP* but purged of *TLP's* residual metaphysics. Whereas *TLP* writes in a metaphysical mode and is only little interested in epistemology (Garver 1989, 101-2), *LSW* approaches many of *TLP's* problems from an epistemological mode in order to provide a logical-linguistic framework for *empirical science* (*LSW* §'s 103, 178). *LSW* (§ 67) replaces *TLP's* LSO's by “the basic elements” of knowledge that “we call *elementary experiences*” [C's emphasis]. For this reason, *LSW* (§'s 58, 66) takes an “auto-psychological” or “methodological solipsism” perspective because it holds that each person's elementary experiences are directly available to themselves but not to anyone else (*LSW*, § 58, 66). This presents Carnap with the difficult problem of showing how an objective empirical science of physics can be grounded in individual subjective experiences (*LSW*, § 59-60, 66-69)!

*LSW* aims to provide “the step-by-step derivation” or “construction” of all concepts “from certain fundamental concepts” needed for a logical-epistemic system capable of completely describing everything that can be said about the world (*LSW*, § 1). *LSW*, like *TLP*, mentions basic un-analyzable objects but since *LSW* (§ 7) takes an epistemological perspective its most basic “un-analyzable objects” are not *TLP*-style LSO's but “‘my experiences’ (more precisely entities which can be called *terms of relations* [emphasis added]).” These basic objects, each person's private experiences, play a very limited role in *LSW's* system: “[T]he basic relations take precedence over the basic elements ... [because] construction theory considers the individual objects as secondary, relative to the network in which they stand (*LSW* § 61). *LSW* (§ 162) does mention “substances” but, following *TLP's* description of the basic substances as “formal,” these are “the unlimited number of possible *forms* [emphasis added] of ordering the elements on the basis of their basic relations”. It is not the basic objects but the “network” of relations that is fundamental to *LSW's* system.

Since *LSW's* substances are demoted to “forms” for “ordering” the relations between experiences, “The basic objects from which all others are constructed are the basic relations [C's emphasis]” (§ 105, Part C. See also § 75). *LSW* (§ 69) continues: “The statements about un-analyzable units ... can only be pure relation descriptions.” *LSW's* simple un-analyzable units dissolve into “pure relation descriptions” (pure formal subjects for the all-important relations). Since *LSW's* demotes its basic objects to formal subjects for the relations, it aims “to characterize all objects through merely *structural* properties ... and to transform all scientific statements into purely structural statements” (*LSW* § 6). *LSW* purports to provide an account of the logical structure of the world not in the limited sense that it provides a framework for describing the structuring of objects needed for a complete description of the empirical world but in the *much stronger* sense that “All statements ... in the constructional system are ... *about nothing but the basic relations* [emphasis added]” (*LSW* § 153)!

*LSW* (§ 177) even states that “Construction theory agrees with *transcendental idealism* [C's emphasis] ... that all objects of cognition are constructed (in idealistic language: ‘are created in

thought’).” This holds even for *LSW*’s (§ 7) most “basic elements” ..., that is, “my experiences which initially have neither names nor properties ... and can be called terms of relations only after certain [mental] constructions have been carried out”. All of these “constructed objects,” including even elementary experiences, “are objects of conceptual knowledge only *qua* logical forms that are generated in a certain way (*LSW*, § 177). *LSW* agrees with *TLP*’s view that the elementary objects are mere formal subjects for the all-important relations, but it arrives there at a very different route. Whereas *LSW* (§’s 82-83, 100), in Kantian language concerning the “construction” or “synthesis” of experience (Cf. Strawson 1966, 93-97), even sees elementary experiences as mental constructions, *TLP* ties *its* simple objects to the truth-functional tautologies that “show [*zeigt*]” the logical form of reality. One cannot get more objective and mind-independent than *TLP*’s view.

Although Stenius (1996, Chap. XI) see a Kantian dimension in *TLP*, it must be remembered that whereas Kant builds his system on synthetic *a priori* knowledge, and, therefore, needs a mind to synthesize the basic experiences into knowledge, *TLP*’s (6.1-6.111, 6.12) system is built upon the “analytic” propositions, the tautologies, and there is, therefore, no obvious need for a mind to “synthesize” the logical structure of the world. Since “the nature of the natural and inevitable ... propositions of logic ... speaks for itself” (6.124), *TLP* needs no mind to speak for them. Whereas *LSW*, like Kant, aims to build a system of knowledge, *TLP* presents a “treatise on logical philosophy” (*Tractatus logico-philosophicus*), not a treatise on mental synthesis. It is Carnap’s *LSW*, not Wittgenstein’s *TLP*, that has this Kantian dimension.

Despite that major difference over the nature of the basic objects, *LSW*’s view is analogous to *TLP*’s view (2.0231) that it is only by the configuration of the LSO’s that the material properties in the world are produced. Just as *LSW* attempts to describe the logical *structure* of the world, not the objects or substances in those structures, so *TLP* attempts to describe the “logical form” (2.18) of reality, not the LSO’s themselves. This is the real significance of Carnap’s title *The Logical Structure of the World*. The basic objects in Carnap’s world, elementary experiences, are mere “terms of relations,” empty subjects for the relations that provide the content for his *purely structural* description of the world.<sup>2</sup>

There is, therefore, a sense in which for both *TLP* and *LSW* the elementary or simple un-analyzable objects drop out of the sense or content of the proposition in favor of the all-important relations (or structure). However, it would be too strong to say that *TLP* is committed to the view that *TLP*’s simple objects in the world do not exist, just as it would be too strong to say that *LSW* is committed to the view its elementary experiences do not exist. It would, for *TLP*, be safer to say that the need to posit simple objects is a formal requirement of logic and leave it at that because nothing more can be said about them (3.221). Similarly, even though *LSW*’s elementary un-analyzable objects appear to dissolve into mere forms created by the human mind, it would also be too strong to say that they do not exist. *LSW* embraces realism in the sense that it embraces the truth of scientific propositions, but *LSW* (§ 52) neither affirms nor denies the metaphysical reality of its elementary objects.

Carnap (*LSW* §'s 12, 16) credits the discovery of the importance of the structural or relational view of the world to Russell's work in the philosophy of mathematics. Russell's seminal insight is that since the mutual relations of points in the Euclidean plane are of the same type as those of the complex numbers, plane geometry, as a branch of pure mathematics, is not concerned with the question what kinds of entities these points really are. For Russell (*POM*, § 27), the "true subject matter" of mathematics is relational types. Russell explains the emphasis that mathematics lays upon relations: "To the mathematician, it is wholly irrelevant what his entities are, so long as they have relations of a specified type. ... [A]n instant is a very different thing from a point; but to the mathematician as such there is no relevant distinction between the instants of time and the points on a line." (*POM*, § 387) As a consequence, Russell says that pure mathematics can "dispense altogether with indefinibles ... because here every concept is defined in terms of general logical concepts," prefiguring *TLP*'s view that the names for simple objects can be eliminated and replaced by purely general logical apparatus.<sup>3</sup> Although Russell's notions of a mathematical relational systems was anticipated by Dedekind (Reck 2020, § 5) and later developed by Cassirer (Friedman 2022, § 3), Russell's *POM* is the seminal work in the logic of relations.

*TLP*'s innovation is to apply Russell's relational model from his philosophy of mathematics more generally to the whole account of "reality". It is one thing to say that the objects of pure mathematics "become variables", quite another to say that the basic "substances" in the real world "become variables," mere "forms" in a pure relational view of reality. But that is precisely what *TLP* does, with the proviso that this "relational" view holds only at the *fully analysed* level. *TLP* does not hold that Norman Malcolm "becomes a variable." Malcolm is a material object and, therefore, a complex structure the complete true description of which at the fully analysed level would require a level of logical analysis that is literally *beyond human abilities* (4.002). *TLP*'s relational view does not apply directly to the sentences of everyday language.

## V.) Reply to the Objection that *Tractatus* Logical Space is not Uniform

What any picture, of whatever form, must have in common with reality, in order to be able to depict it—correctly or incorrectly—in any way at all is logical form, the form of reality

Wittgenstein, *Tractatus* (2.18)

One might argue that *TLP*'s (2.171) implication that different kinds of pictures can have different "forms", that "A spatial picture can depict anything spatial, a colored one anything colored, etc.," suggests that *TLP* sees logical space as divided into different regions with different kinds of simple objects with different kinds of logical form, color form, sound form, etc. In this case the claim of the present paper that there is an analogy between *TLP*'s notion of logical space and internally indistinguishable points in Euclidian space is wrong.

This objection is misguided. Note that 2.15-2.171 is concerned with "pictorial form." The

notion of logical form is not introduced until later at 2.18 where *TLP* clarifies that despite the fact that different pictures can have different kinds of *pictorial form*, color form, spatial form, etc., *all* pictures must, in the final analysis, possess “logical form, i.e., the form of reality.” Thus, *TLP* is in 2.15-2.18 making the distinction between mere pictorial form, which might, for example, be mentioned in a lecture to art students, and logical form, which latter belongs is a philosophical concept generally unfamiliar to most non-philosophers but which must be possessed by *all* pictures if they are to represent reality at the fully analyzed level. The art student is only concerned at the everyday level with the pictorial form of the picture, e.g., they must choose the right colors to represent a rainbow. Their drawing of a rainbow must *also* possess logical form but that is irrelevant to their everyday concerns and generally unknown to them. Thus, 2.171 has *nothing* to do with *TLP’s* philosophical point at 2.18 that at the *fully analyzed level* propositions must possess logical form if they are to represent reality either “correctly or incorrectly”. There is no suggestion that *TLP’s* logical space divides into different regions with different kinds of logical forms.

Further, Wittgenstein only began to consider the possibility of such different regions of logical space many years later, after he returned to philosophy from a long absence, in his 1929 “Some Remarks on Logical Form” and his 1930 *Philosophical Remarks* with realization that *TLP’s* notion of logical space was inadequate. Whereas *TLP* (4.211, 5.124) holds that all the elementary propositions must be logically independent of each other, Wittgenstein now begins to take seriously the idea that there are some *systems* of elementary propositions in which propositions are incompatible with each other, e.g., “a is red” and “a is blue” are logically incompatible even though neither is the truth-functional negation of the other. Thus, Wittgenstein begins to consider replacing *TLP’s* (2.1512) view that a proposition “is laid against reality like a measure” with the new idea that it is “systems of propositions”, e.g., the *system* of color propositions or the system of sound propositions, etc., within which there are *internal elementary non-truth-functional incompatibilities*, that is laid against reality. This, however, is a significant modification that initiates the unraveling of *TLP’s* logical system. The fact that Wittgenstein only begins to consider this subversive idea in these later works shows that it had *not* been present in *TLP*.

## **Conclusion**

*TLP’s* view that the world can be *completely* described by fully generalized propositions in which no name is correlated with any object means that *TLP’s* notion of LSO’s and the simple “names” for them in elementary propositions are part of the “ladder” that must be thrown away after one has climbed it. Just as it does not matter to Russell what mathematical objects “really” are, if anything, it does not matter to *TLP* what its “simple objects” really are, if anything. Since *TLP’s* simple objects are only the formal subjects of propositions, its elementary propositions can only say how those “colorless” formal subjects are related, nothing more. *TLP’s* internally indistinguishable LSO’s *can* be distinguished by employing *fully generalized* propositions used to describe how these formal subjects are contingently configured. Carnap’s *LSW* is a further development, with some illuminating

differences, of *TLP's* notion of logical space as a relational (or structural) system. The misplaced focus on *TLP's* notion of LSO's distracts from the fact that *TLP's formal notion of LSO's as like* the intrinsically indistinguishable points in geometrical space thereby makes a novel departure from the substance metaphysics that has dominated Western philosophy since Aristotle to a new model on which "reality" consists entirely in systems of relations.

### Endnotes:

1. References to Wittgenstein's *Tractatus (TLP)* are to proposition number, to his *Notebooks (NB) 1914-16* by page number, to his *Philosophical Remarks (PR)* by section and paragraph number.
2. Carnap does distinguish between relation-descriptions and structure-descriptions, the latter forming a "higher level of formalization and dematerialization" (*LSW's* 11-12, 153), but this goes beyond the primary focus here on *TLP's* LSO's.
3. Russell elsewhere in *POM* seems, inconsistently, to deny that numbers can be defined purely relationally (Pakaluk 1992, 303).

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