Induction and Probability

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Abstract:

The present research aims to examine the different accounts of induction given by Aristotle, Hume, Leibniz, Carnap and De Finetti, trying to support that probability calculus offers a sufficient grounding of inductive logic. The term induction had been contrasted to deduction, by Aristotle. The Neoplatonic philosopher Alcinous suggested that dialectic firstly investigates the substances and then the accidents. There are five kinds of dialectic reasoning: division, definition, analysis, induction and syllogistic. The first three concern with substances, the last two with accidents. Although Leibniz regarded probability theory as a basis of inductive logic, Hume's skepticism was seminal for the reappraisal of the role of induction in modern philosophy. Enhancing Hume's criticism, Popper and Wittgenstein completely denied that scientists use induction. Hans Reichenbach, however, attempted to build a theory of justification for the use of induction, based on a factual basis of other successful predictive methods that make induction feasible (Earman & Salmon 1999). Moreover, Buchdall (1969) stressed that we must distinguish the inductive process of the scientist from the inductive conclusion, which comes after the completion of observation and experimentation.

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Hume's Legacy

Hume wrote that we have reason to believe in the truth of a state of affairs only if we can connect it with something we now perceive or remember: though our empirical inferences carry us beyond our memory and senses, and make us certain of matters of fact, which happened in the most distant places and at the most remote times; yet some facts must always be present to the senses or memory, if we may in the first place be able to draw those inferences. As is the case when we start from archaeological findings and follow up with careful study to reach the eyewitnesses of the past. In short, if we did not take as our starting point some fact, which is present in our memory or senses, our reasonings would be merely hypothetical, the chain of our conclusions would have no support, no foundation.[1]

Hume's thought on the problem of induction focused precisely on the absence of such support, mainly regarding future events, according to the following argumentation: All reasonings may be divided into two kinds, namely, demonstrative reasonings concerning relations of ideas, and moral reasonings concerning matters of fact and existence. All arguments about existence are founded on the relation of cause and effect. Our knowledge of this relation is derived entirely from experience.

The two propositions which follow are far from being the same proposition, 'I have found that such an object is always followed by such an effect', and 'I predict that other objects, which are, apparently, similar, will be followed by similar effects.' All our experimental conclusions have for their starting point the assumption that the future will be conformable to the past. The problem of causal arguments lies in the observation that if the course of nature changes, all experience becomes useless, and the past cannot be a rule for the future and can provide no basis for any inference. No empirical argument can prove the similarity between the past and the future.[2]

Self-evident inductive connections are not rationally justified, because the inference of an event B from another event A is never demonstrative, since it is only an empirical proposition and therefore, we can deny it without contradiction; there is no synthetically necessary connection between the events A and B; the only reason to expect B given A is past experience, but the inference is invalid because it is based only on experience. It is an inductive leap. To support induction, we would need an additional premise: 'examples of which we had no experience will resemble those of which we had', but all we can do is detect and conceive of a change in nature, however, even the possibility of such a conclusion is based on the assumption that nature is governed by uniformity, so we look back to the past. With this double questioning of both the present and the past we conclude that the general principle of the uniformity of nature does not apply. So, since our conclusions are not formally valid and since their conclusions are not possible without circularity, we consider that they are not rationally justified but are based on habit alone.

Our bewilderment before the infinite nature of the recursion in the chain of causal connection, our inability to attribute the cause of every cause seem insurmountable obstacles. For this reason, Hume suggested that simply the repetition of a certain act or operation produces a tendency to renew the same act or operation, without being compelled by any reasoning or by any mental process. Hume contends that this tendency is the result of Custom, which is not, however, an ultimate cause, but only a principle of human action that becomes known from its effects.

We cannot observe anything beyond a continuous succession of objects, while the secret forces, by the aid of which all natural functions are performed, never appear to the senses, because elasticity, gravity, cohesion, momentum have not yet been adequately studied. We must of course note that fire and heat, snow and cold are certainly not connected with each other by habit, as Hume maintains. This is an extremely paradoxical and contradictory assertion, where his thought engages in some kind of Psychologism.

But the most interesting point of the Humean critique is the concept of moral reasoning. I believe that it indirectly refers to a paper published in 1738 by Daniel Bernoulli, in which he solved the St. Petersburg paradox. Bernoulli distinguished two concepts of expectation: mathematical and moral, which considers the individual characteristics of the subjects who take risks in a game. In the case of the paradox in question, moral reasoning refers to economic utility: infinite increases in utility are directly proportional to infinite increases in wealth and inversely proportional to the amount of initial wealth. This idea of Bernoulli had an enormous impact on the science of probability and on the social sciences.

Debugging Hume's Error

There is no reason to doubt that the relations of ideas are a priori analytic propositions. But there are serious objections to Hume's view that only the relations of ideas are demonstrative propositions and that all the propositions of Geometry are relations of ideas, that is, they do not depend on what happens anywhere in the universe. The shift of the scientific horizon from the closed world to the infinite universe has made the latter claim false. It turns out that geometry has empirical foundations and many, like Mill, argue the same for arithmetic. But my objection is deeper and refers to the first claim and to the concept of proof in Hume. I also comment on his epistemological resort to the concept of habit.

Let the sentences π_0, π_1, π_2, π_3 of our language be:

π_0: I observe the sky from the highest peak of Naxos.
π_1: There are absolutely no clouds above and around Naxos.
π_2: We will not have rain in Naxos soon.
π_3: I can, with the help of meteorological radars, searchlights and EMY balloons, determine what I mean by 'soon'.

Aristotle would characterize the proposition π_1 as a presumption of reasoning, an average that makes the reasoning valid, precisely because i) it is connected to the specific relations of part and whole with the extreme terms but also ii) because it is the empirical presumption of the reasoning

$$\pi_2 \subset \pi_1 \subset \pi_0$$

in which the term π_2 is a subset of the average term π_1; the average term π_1 is also a subset of the term π_0 (the proposition π_3, which can be understood as auxiliary, is not included). For short-term forecasts derived only from local observation, clouds are one of the two most important available factors.[3]

Remaining in our position, which ensures satisfactory visibility and observing the sky, we may later see at 30,000 feet some cirrostratus that will cast a white veil around the sun or some sparser cirrocumulus. Only a sudden evaporation and convergence of air currents and a sudden and vertical ascent of warmer air to a higher point, usually riding a cold current, either climbing a high mountain, or colliding with another air current, can create clouds or carry some. But even then, we may know that it will not rain soon, if there is nothing beyond cumulus, the charming wavy white clouds of summer (fair weather clouds).

The problem of induction

According to Aristotle, the inductive arguments demonstrate the universal, because they show that the particular is evident, obvious. They use examples for this purpose. For the universal exists, when it is demonstrated on the contingent and first instance. Induction focuses on the most probable, the clearest, the most familiar and common to many. On the other hand, deductive reasoning is more hasty and more active towards contradiction. These may simply be reminiscences, where we do not examine the actual reality of what is being discussed.

However, induction par excellence is not demonstrative, e.g. the inductive reasoning:

the α_i are b
the α_i are c
therefore all c are b

is not demonstrative. What are the characteristics of a demonstrative procedure, according to Aristotle? First, the only valid generalization scheme is the reasoning that displays a universal quantifier, categorical or negative, in the major term and an existential or indefinite[4] categorical in the minor. All other syllogisms containing a universal quantifier in other positions are invalid or not perfect. But even then, the calculation may be inaccurate or impossible to be accurate. Infinity cannot be exhausted.

If we cannot calculate the size and height of the clouds, that is, if their lowest point reaches two miles from the ground, while the highest reaches five miles (cumulonimbus: thunderhead), then not only will we fall outside the induction, but we will also suffer a sudden storm.[5] If we were to give a more formal character to the propositions π_0, π_1, π_2, referring, for example, to mathematical meteorological data and to distances of latitudes and longitudes, to curves of the earth's surface and to heights above sea level, our reasoning would be universal. That is, it could also admit of inductive generalizations, considering longitudes and latitudes, curves and heights as variables that take on various values. However, it seems that these generalizations, while more formal, would only offer us high degrees of conviction and would remain plausible; they would not be pointing-out, as Aristotle supports, that is, they would not be certain propositions.

In any case, we can only ensure satisfactory predictability when our propositions are indicative, referring to empirical data, as Hume would say. And we do not know whether we would ever obtain certainty for exceptional cases, as for example when approaching the Himalayas or in the tropical cyclone regions, namely the West Indies (hurricanes), the East Indies and Japan (typhoons), Australia (willy-willies) and the Philippines (baguios), where cyclones reach 136 miles per hour and make our forecasts extremely uncertain, especially as regards the time parameter, since the monthly frequency of tropical cyclones in the North Pacific is $f \leq 4.5$ in September. But in the Mediterranean the strongest winds are the Bora and the Gregale, which do not exceed 82 miles per hour.[6]

The problem of proof comes back more sharply at this point. We observe that formality in the use of language reduces provability. Hume may have been refuted as can be seen from our fair-weather example. However, the skepticism that he developed with unparalleled skill in his work was very fertile. An empirical rational epistemological position is based on sharp criticism. But it should not underestimate induction, to end up in the deification of the deductive method. Do we prefer the distant formality that does not find ways of development and grafting into experience?

Only after Bacon do we overcome the stage of thought experiments and set observation and experiment as prerequisites for the construction of scientific theory. Induction is poor if it reaches the principles of science by simple enumeration without using exclusions and conclusions or appropriate analyses of nature, according to Bacon. He suggests also that the method that axiomatizes is a source of errors. The first aim of true induction is the rejection or exclusion of individual natures that are not found in an example in which the given nature is present or are found in an example from which the given nature is absent or are found to increase in an example where the given nature decreases or to decrease when the given nature increases. Bacon's analysis of induction was fruitful in the organization of scientific practice, because it highlighted and emphasized old and new methods that renewed science, such as harvesting, tables, rejection-exclusion, classification of examples, privileged examples, supports of induction, limits of research, preparation of research.[7]

Subset or implication?

Let the sentences π_4, π_5, π_6 of our language be:

π_4: I observe the sky while flying with a helicopter over Greece.
π_5: There are absolutely no clouds over and around Greece.
π_6: We will not have rain in Athens soon.

And here our reasoning has the form

$$\pi_6 \subset \pi 5 \subset \pi 4 \text{ assumption}$$
$$\pi_4 \wedge \pi_5 \wedge (\pi_5 \subset \pi_4) \text{ assumption}$$
$$\pi_4$$
$$\pi_5$$
$$\pi_5 \subset \pi_4$$
$$\pi_5 \wedge \pi_6 \wedge (\pi_6 \subset \pi_5) \text{ assumption}$$
$$\pi_5$$
$$\pi_6$$
$$\pi_6 \subset \pi_5$$

Our reasoning would be characterized as perfect, insoluble and true by Aristotle, because it conforms to the first, the only valid of the three schemes of logical inference. In fact, it is a proof, stemming from true and first and immediate and more familiar and a priori and causes of the conclusion.

In this proof, neither the concept of habit, nor any related concept, plays any role at all. Our proof has a clear and pure empirical character, while the concept of habit is an unverifiable hypothesis that undermines our empirical intentions.

In the case where one of our propositions is not connected by a part-and-whole relationship with each of the other two propositions, we end up with an invalid reasoning. The reasoning is also true if its terms are true. Since the exclusive disjunction does hold:

$$(\pi_5 \subset \pi_4) \vee (\pi_7 \subset \pi_4)$$

where π_7: There are clouds over and around Greece.

We prefer to express the evidential relation through the subset relation, because the truth table of material inference is loaded with errors: for values F ⇢ T it gives a truth value T in the conclusion, which is not sensible and logical for empirical induction. Let us recall at least one case from our everyday life that we would like to risk using inferences containing false premises.

In our real earthly world, the implication F ⇢ T is not true, while in material implication there is no 'real connection' between the premises and the conclusion. We simply assert that it is not true that the premises are true when the conclusion is false.

Copi[8] distinguishes material implication, which is empty of content, because the premises may be unrelated to the conclusion or refer to non-existent objects,[9] from implications that express logical connection, definition, cause and decision.

The truth table of implication has the following correct form:

$$(T \dashrightarrow T) \leftrightarrow 1 \dashrightarrow 1 \leftrightarrow T$$
$$(T \dashrightarrow F) \leftrightarrow [T \dashrightarrow (1\text{-}T)] \leftrightarrow 1 \dashrightarrow 0 \leftrightarrow F$$
$$F \dashrightarrow T \leftrightarrow [(1\text{-}T) \dashrightarrow T] \leftrightarrow 0 \dashrightarrow 1 \leftrightarrow F$$
$$F \dashrightarrow F \leftrightarrow [(1\text{-}T) \dashrightarrow (1\text{-}T)] \leftrightarrow 0 \dashrightarrow 0 \leftrightarrow T$$

Thus, we can define the subset relation as the inverse function of the inference relation, that is,

$$(\rho \dashrightarrow \sigma) \leftrightarrow (\rho \supset \sigma) \leftrightarrow (\rho \subset \sigma)^{-1} \leftrightarrow (\sigma \subset \rho)$$

Indeed, it is true that sentences with truth value T are subsets of sentences with truth value T. It is also true that sentences with truth value F are subsets of sentences with truth value F. However, it is neither true that sentences with truth value T are subsets of sentences with truth value F nor that sentences with truth value F are subsets of sentences with truth value T.

De Finetti[10] identifies the concept of subset with logical implication (not with material implication) and distinguishes the implication of an event A from another event B

$$A \subseteq B \ .=. \ \vdash A \leq B$$

from the specific equality of A and B

$$A \equiv B \ .=. \ \vdash A = B$$

and from the strict implication

$$A \supset B \ .=. \ \vdash (A \subseteq B) \land \sim (A \equiv B)$$

The concept of subset can be linked to terms such as probability, informativeness and preference. Probability theory is based on the concept of a set. Events or contingencies are subsets of the sample space δ. The outcome of a chance experiment is a simple event or a simple contingency. A set of simple events is called an event or contingency. If our sample space is discrete, that is, countable, either finite or infinitely countable, all its subsets correspond to events and vice versa. If it is continuous, only some subsets correspond to events.

In our thought experiment, we consider that π_2 offers us less information than π_1, just as π_1 offers us less information than π_0, and correspondingly, even when we are not sure (that is, although we do not have a confidence level of 1) we can nevertheless derive the valid formula

$$P(\pi_2) \leq P(\pi_1) \leq P(\pi_0)$$

Also, if there are indeed no clouds in the sky, we believe that π_6 is more preferable than π_7, because with these data we have, accepting it involves less risk.

The empirical confirmation of propositions π_6 and π_7 can be regarded as a preference issue, precisely because these propositions are not causally connected to each other.[11] In the case where there is a causal relationship between two propositions π and ρ, then we compare them based on the information they contain and the proposition that is more informative is considered the cause of a proposition with a smaller information size.

Hempel on confirmation and induction

In 1945, in his article *Studies in the Logic of Confirmation*, Hempel tried to prove the logical invalidity of Nicod's confirmation criterion that i) an object x confirms a universal hypothesis if and only if it satisfies both the antecedent and the subsequent. ii) An object x disproves (disconfirms) a hypothesis if and only if it satisfies the antecedent but does not satisfy the subsequent proposition of the hypothesis. That is, the proposition

$$\forall x \ [\pi_i(x) \supset \pi_j(x)]$$

is confirmed if and only if x is π_i and x is πj and is disproved if and only if x is π_i and x is not π_j. Hempel adds that Nicod implies that the two other cases, namely when x is not π_i, are neutral or irrelevant to the hypothesis.

So let the sentences

$$S_1: \forall x\ [raven(x) \supset black(x)]$$
$$S_2: \forall x\ [\sim black(x) \supset \sim raven(x)]$$

i.e. S_1: 'All ravens are black' and S_2: 'What is not black is not a raven'.

Hempel observes that the fact a: 'black raven', N-confirms[12] S_1 but is considered N-neutral[13] for S_2.

However, as Hempel himself claims,

i) the fact a also confirms S_2. Similarly, the fact d: 'neither raven nor black', while confirming S_1, is not N-neutral for S_2. On the contrary, it also confirms S_2.

ii) The propositions S_1 and S_2 are logically equivalent, that is, they are different formulations of the same hypothesis. This is the equivalence condition: Whatever confirms (or unconfirms) one of the two equivalent propositions confirms (or unconfirms) the other. Here is the important problem that Hebel identifies: that the Nicod criterion violates logical equivalences and makes validation dependent not only on the content of the hypothesis but also on the formulation.

Nonetheless, a member of the audience may argue, that this is due to Hempel's vagueness, not only Nicod's. Two types A and B are logically equivalent if and only if the equivalence $A \leftrightarrow B$ is a tautology. But $S_1 \leftrightarrow S_2$ is not a tautology. For example, the proposition

$$(\pi \supset \rho) \supset (\sim\rho \supset \sim\pi)$$

is indeed one of the axioms of Gottlob Frege's *Begriffsschrift*, yet it is not an instance of logical equivalence but of material implication and is moreover not quantified.

Propositions S_1 and S_2 differ from a specific perspective. The second intervenes in the field of definition and amplifies the definition of raven, implying that objects that have exactly the same specific features as ravens, but are not black, aren't ravens either. If we include the color black in the definition of raven, it will not be possible that 'when ravens are not black, they do not look like ravens.' Whereas if the color black is not part of the definition, it will be possible that 'ravens that are not black do not look like ravens.' An extremely important distinction, if we want to discover them and not miss them. And it is also possible to arrange for cases such as the one that makes the nymph not looking like a butterfly, although it is going to be one.

In another instance, Hempel had really observed that there are no inductive rules. We invent our hypotheses and theories; we do not produce them. Scientific knowledge is not the application of some inferential inductive process, but formulates inventive hypotheses, gives bold answers to problems, and makes experimental implications.[14] Our suspicious member of the audience could have therefore considered S_2 as such a bold attempt.

Confirmation, conjunction and disjunction

We must remark that Hempel, by rejecting the Nicod criterion, refuses to limit the content of confirmation to the context of conjunction. For if the equivalence of S_1 with the proposition S_2 holds, then we cannot formulate any safe confirmation rule based exclusively on conjunction relations, since, as he claims, i) 'white shoes' confirm S_1 and ii) 'black ravens' confirm S_2. Thus, Hempel a) considers that irrelevant or neutral evidence can be factor of confirmation and b) highlights the case of cancellation (T⇢F ⇢ F) through disjunction, thus returning to the criterion of inductive rejection or exclusion formulated by Bacon. Hempel's contribution is the emphasis he gives to the differentiation of cases.

Nevertheless, of the two controversial examples, i) seems unfortunate. It sounds not logical that 'white shoes' should confirm an irrelevant proposition, because the amount of relevant information is zero.[15] But are other propositions, like S_2, confirmed by corresponding examples? If we are to distinguish cumulonimbus from cumulus or other clouds, the proposition k: 'Every time it does not rain there are no cumulonimbus,' seems to be strengthened by instances of cumulus and cirrocumulus, but stratus, altostratus and nimbostratus which bring rain are neutral with respect to k. Hempel's criterion is very useful for disjunction of instances of a class. However, we must exclude expressions that can only result in confusion, such as 'every non-raven confirms S_1 and S_2' or 'that all mermaids are green'.

The example ii is more tempting and seems to confirm S_2. The examples we have of 'black ravens' are so numerous that, since we have not yet discovered a raven that is not black, we believe that S_2 is confirmed by the 'black ravens' we have observed. But this too presents difficulties. In ancient times they did not know that there were black swans. A white swan seemed to confirm the proposition 'What is not white is not a swan'. When we discovered black swans in Australia the opposite was proven. Probably, we may never find raves that are not black. But even then, we cannot be sure.

Some shades of black may overlap with colors or with gradations of the achromatic scale white, gray, black. And while a 'gray-blue raven' probably disconfirms both S_1 and S_2, it is not certain that gray-black, black-brown or black-blue confirm S_1, but they seem to confirm S_2. Unless we formulate them more clearly. But Hempel denies that the formulation is essential. Certainly, the formal formulation is less essential than the physical scientific one: We would then say that 'every raven is covered by plumage whose surface scatters almost no incident light, i.e. the diffusion coefficient, expressed by the quotient of the scattered light by the intensity of the incident light, is approximately zero.'[16]

Moreover, we can show with absolute clarity that S_2 is not equivalent to S_1 by examining the sentences S_3 and S_4. That is,

$$S_3: \exists(x)\ [\sim black(x) \supset \sim raven(x)]$$

$$S_4: \exists(x)\ [black(x) \land raven(x)]$$

The sentence S_4: 'There is at least one object x that is black and is a raven' does not imply that the property 'is black' has any causal relation to the entity raven. It is purely a matter of preference whether to consider ravens to be black.

But with the symbol of implication that we used in S_1, we denoted a causal and necessary relation. Unless we followed Russell and read the universal quantifier as 'every'. Salmon[17] also, based on the rejection of necessity, supports a physical and not a logical or metaphysical concept of causality. The point of departure from the ancient conception of induction and causality is the rejection of necessity. By saying 'Every raven is black' we give appropriate emphasis to the fact that we are expressing observational data.

What about proposition S_2? We can also read this as 'Anything that is not black is not a raven', but we will not have escaped the scope of the proposition 'Everything that is not black is not a raven'. The problem is that the natural language propositions with which Hempel primarily translates the formal expressions S_1 and S_2 are not equivalent, because they do not both use the same expression for the universal quantifier. 'Everything' is not equivalent to 'anything'.

And neither of the two sentences satisfactorily translates the symbol of implication. Suppose that S_1 corresponds to the statement that the color black is a subset of every sample we have of ravens. What meaning should we give to the implication we make with S_2? In what way does the property 'is not black' imply the entity 'is not a raven'? The claim that beings that are not ravens are a subset of beings that are not black is clearly false, whether for all or for each. Only some of the beings that are not ravens are a subset of beings that are not black. The error arises from the binding of the variable x with the universal quantifier. That is why we use the existential quantifier in S_3 and S_4. Regarding the hypothesis that the color black has some causal relationship with the entity 'raven', it can be posed as a scientific question. Until it is answered, only S_4, which uses the conjunction relation, is correct.

Confirmation and equivalence

We might ask ourselves - if we had to choose - which proposition best validates the empirical fact 'black raven'? S_1, S_2, S_3 or S_4? Obviously, S_1 is confirmed by the class of the points of the probability space $P(S_1)$, where we place each black raven. S_2 is weakly confirmed in the complement of $P(S_1)$, in the probability space $P(S_2)$, namely Ω-P(S1).

S_2 does not express a sequence, like the sequence of observed black ravens, which we can approximate from the statistical frequency of the data and efficiently reconstruct by data feedback. S_2 expresses an uncountable and continuous function of the events that are not black and are not ravens at points in $P(S_2)$ that correspond to probabilities.

$P\ (S_2)\text{: } \Omega - P\ (S_1)$

Not every point of Ω-P is a subset of P(S_2), because a) there are events without color and colorless objects. And while black, gray and white belong to the achromatic scale of optics, there may be colorless objects that do not belong to this scale, b) there may be objects that 'change color' and c) there may be objects that are ambiguous whether they are black and d) there are an infinite number of clearly non-black objects and, plausibly, some of them may be a raven. Thus, we conclude that the simple fact 'black raven' satisfactorily confirms only the proposition S_5, which does not express an implication but the unspecified fact that

$$S_5\text{: black}(x) \subseteq \text{raven}(x)$$

'objects x that are black belong to the concept raven'. S_1 is satisfactorily confirmed only by the entire series of ravens and if they are black. However, it is not verifiable. If we ever find ravens of a different color, or better yet colored ravens, or gray or white, and their frequency is, let's say 0.1, then S_1 and S_2 will be refuted.

Belief

Logically equivalent expressions exist in the field of logic and in any formally axiomatized language. However, in the experimental laboratory, it is very likely that the data can refute the hypotheses of scientists, and the most vulnerable from this point of view are the formal expressions. But the same thing happens at the level of natural observation, as in everyday life, where we do not formulate scientific hypotheses but simply form beliefs.

Let us suppose that a merchant who runs a stationery store order pens from the POP company. However, it happened that repeated errors were made in the execution of the order and that they sent him all the colors of ink, except black. The result is that the merchant forms the belief that 'the POP company does not produce black pens.' Because not only in the first order but also in the other three that were made for correction, they did not send him a single black pen. We thus find a good example to show that twisted expressions like 'Everything that is not black is not a POP pen' are completely inappropriate when one must make decisions. We realize that such a twisted logical expression cannot describe the merchant's dilemma.

He will risk rejecting POP not because he is certain of some belief about POP, but because he is not certain of anything. And he estimates that uncertainty is harmful. His conclusions and actions, in any case, will express his tendency to recover the coherence of his activity. He is motivated by uncertainty, not quite the same as De Finetti's[18] view, who assumed that we do not want to participate in bets that will certainly result in our loss. A set of our predictions is therefore coherent if among the combinations of bets there is none for which the odds are always uniformly negative.

Let us return to the earlier distinction between experimental activity, physical observation, and the formation of beliefs in everyday life. Often the validation of scientific or technical hypotheses encounters insurmountable obstacles. How can a consumer verify whether a piece of meat or a quantity of sausages is fresh, since their red color may be the result of the action of sodium nitrite? It is very doubtful that he will be able to determine whether this sodium salt is present in the meat by the method of testing that Hempel reminds us of. Sodium vapor does indeed color the flame a bright yellow, but we will only test the meat if we have somehow formed the belief that sodium salt is possibly in it. We do not test for sodium like that in the comfort of our own homes. How can a consumer perform such an experiment? And, even if he does, he still won't be sure that it's sodium nitrite.

An alternative way is to make another disjunction and check another property of fresh meat, such as smell. Even if the consumer had the belief that it is impossible to find sodium nitrite in meat and his belief was justified, it would still not be true because it would be impossible to confirm the universal generalization that 'no meat and no sausage has sodium nitrite'.

And there would be no confirmation, because we would not be able to check all the products one by one and above all we would not be sure that no one would ever try to deceive us in this way. The best we could achieve would be to ensure many and effective health checks and relative degrees of conviction or trust, but not certain conviction. We conclude that there are many obstacles to effective confirmation.

Here we find the starting point of the concerns raised by the subjective interpretation of probabilities, when it establishes that, although its opponent, the objective interpretation, uses rules that limit the frequency of incorrect decisions to a fixed limit, regardless of values and unknown parameters, it cannot effectively apply these rules to the control of, for example, production. As De Finetti[19] argues, every control can be done incorrectly, and no one can be satisfied with the fixation of the production of defective products even at thresholds of the order of 0.01%.

Conclusions

Given his initiative to upgrade the applicability of formal equivalence, it becomes clear that Hempel wants to keep his distance from observation and experiment. While he considers the criterion of logical equivalence of formal expressions to be their confirmation or disconfirmation by the same sets of observational data, he is not prepared to upgrade the role of empirical control. For example, he contends that most scientific hypotheses and laws express regular connections of characteristics that are not observable in the sense of direct observability.[20]

Nevertheless, the observation, as by Pavlov, of the psychological connection and dependence of the bell stimulus on the hunger stimulus, as well as everyday events such as our observation of the wear of car tires while driving, testify that knowledge is formed by observation, and even by direct observation.

This happens even with the most complex empirical phenomena, for example with events such as the observation of the formation of violet light, violet blue, reddish violet from blue and red photons, which on surfaces of a certain thickness are strongly reflected one or the other or both. On observations we base all our verifiable explanations, such as the scientific explanation of the creation of the feeling of black from the observation of the incidence of light in areas that cancel any reflection, which is accompanied by the simultaneous descent of the curve of both blue and red and their simultaneous arrival at their minimum points. Established in direct observation are also the findings concerning the operation of the chronometer in the above phenomena, that it turns faster when we observe the blue photon and slower when we observe the red.

With the concept of blue and red 'photon' we describe precisely this difference in the movement of the chronometer.[21] Hempel insists on the conceptual character of scientific knowledge and, for this reason, tries to subject confirmation to purely logical criteria. He divides, in principle, confirmation into the following acceptable cases: entailment condition, consequence condition, based on the concept of class, consistency condition and equivalence condition. And finally, he identifies the confirmation of a hypothesis with the *satisfaction* criterion of confirmation, that is, he identifies it with the logical inference from an observational report of the development of the hypothesis in question for the class of objects referred to in the observational report.[22]

Indeed, we form the concept of class through experience. Our knowledge is based on observation. We first perceive a change in nature, as Hume would say. And from the repetition of this change, we form the concept of event. With the repetition of the same event, we first observe the difference between the same and its absence. Observing that this difference of appearance and absence returns and identifying long intervals of absence, we finally form the concept of negation. And, with feedback again from observation, we distinguish simple events from conjunctions of events and formulate conjunctions and disjunctions of events. With even more intense effort, we distinguish between conjunctions of events, those that are conjunctions of similar events, that is, which constitute sets of similar events that we consider as facts.

Thus, with new repetitions, we form the concept of similarity and may make a hypothesis, which if confirmed will take the form of implication. We gradually learn to make successive confirmations and disconfirmations and from a very early age we apply the act of comparison and especially comparison based on probability. A topical problem that we raised at the beginning of our work concerns not only the truth table of implication but also more generally the persistence of logical analysis in the relation of implication, with a parallel degradation of the relations of negation, disjunction and conjunction, that is, of the primal nature of comparison.

If conjunctions are the enemies of high probabilities, disjunctions are their indispensable allies, as Salmon observes.[23] However, when he refers to 'conjunctions' and 'disjunctions', he means logical relations between events that are equally probable, that is, he refers to phenomena that exhibit a statistical regularity, to which Jacob Bernoulli's limit theorem applies, which legitimizes the practice of equating statistical frequencies and probabilities.[24]

By contrast, if we insist on the opposition of statistical frequencies and probabilities, we will adopt the subjective interpretation of probability and will make yet another distinction between statistical probability and logical probability.[25] Carnap, for example, bypasses Bernoulli's theorem, which concerns cases whose number tends to infinity, and, rather wanting to include cases with finite number of samples, states the principle of indifference or insufficient reason: that if we do not know any reason to explain why one situation occurs instead of another, then these situations are equally likely.[26]

The subjective interpretation clearly distinguishes between accepting a hypothesis and decision and refers to cases of known a priori probabilities. The objective or statistical interpretation is concerned with conclusions that remain true regardless of a priori probabilities. Intermediate theories, such as Carnap's or Kendall's, accept neither the formalization of statistical computation theory nor the indispensable character of a priori probabilities.[27] This is why Carnap claims that a priori probabilities are uniformly distributed.

Finally, Hempel maintains a subjective position, and this is evident from his emphasis on the equivalence condition. For the subjective theory there is no stability of probability or independence of the probabilities of the tests, because there are no unknown probabilities as in statistics, but only 'equivalence' or 'exchangeability' between known probabilities that differ in the number of favorable cases.[28] Any hypothetical probability of a test remains subjective and depends on the chosen opinion, since the assessment of a subjective probability concerns a single test. On the other hand, frequency functions form a family of one parameter, with independent variables referring to a sequence of independent tests. A priori probabilities are an important and useful concept, especially for the social sciences. But in the natural sciences the objective interpretation of probabilities is much more applicable and effective.

Endnotes:

1. Hume, *An Enquiry.*
2. *Ibid.*
3. Donn, *Meteorology,* pp. 97-98.
4. An indefinite term is one that is neither universal nor particular.
5. Donn, *Meteorology*, p. 102.
6. Strong cyclones are created not very close to the equator but in the tropics, where a) there are sufficient Coriolis forces and b) the evaporation of water is easier, because its temperature is greater than 270 C (Donn, Meteorology, pp. 227-228).
7. *New Organon.*
8. *Introduction,* pp. 223-227, 243-244.
9. As Hempel's example shows, $\forall x$ [mermaid (x) $\supset$ green (x)] (*Aspects of Scientific Explanation,* p. 16).
10. *Theory of Probability,* 40.
11. Maher, 1-33.
12. N-confirms: confirms according to the Nicod criterion.
13. N-neutral: is neutral according to the Nicod criterion.
14. Hempel, *Philosophy of Natural Science*, pp. 15-17.
15. Salmon, *Causality and Explanation,* p. 98.
16. Alexopoulos, *Optics,* p. 226.
17. *Causality and Explanation,* p. 24.
18. *Theory of Probability, A critical introductory treatment,* Volume 1, p. 76.
19. *Recent Suggestions for the Reconciliation of Theories of Probability,* p. 214.
20. *Aspects of Scientific Explanation,* pp. 25-29.

21. Feynman, *QED.*
22. *Aspects of Scientific Explanation,* pp. 35-39.
23. Salmon, *Causality and Explanation,* pp. 97-98.
24. Daston, p. 31.
25. Carnap, An *Introduction to the Philosophy of Science,* p. 22.
26. *Ibid*., p. 23-24.
27. De Finetti Bruno, *Recent Suggestions for the Reconciliation of Theories of Probability,* pp. 218-221.
28. De Finetti Bruno, *Theory of Probability, A critical introductory treatment,* Volume 1, p. 7.

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